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Free vibration characteristics of cylindrical shells partially buried in elastic foundations

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Abstract

Free vibrations of cylindrical shells partially buried in elastic foundations based on the finite element method were examined. The shells are discretized into cylindrical finite elements and the distribution of the foundation in the circumferential direction is defined by the expansion of Fourier series. The present formulation can be simply applied to consider non-uniformities in the foundation both in the circumferential and longitudinal directions. Convergence issues with the present method are explained. Numerical results of the natural frequency and mode for various shell geometries and foundation parameters are given to provide a clearer picture of the shell characteristics in linear vibrations. The relative stiffness ratio of foundation and shell is discussed. The results of free vibration analysis for partially suspended shells on elastic foundations are also presented.

1. Introduction

Cylindrical shells are widely used in engineering in the form of structural components for pressure vessels, storage tanks, pipes, water ducts, process equipment and in other applications. For some cases, these shells are laid on a soil medium as the foundation.

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Yang et al. [1] investigated the behavior of whole buried pipelines subjected to sinusoidal seismic waves by using finite elements. Free vibrations of whole buried cylindrical shells on Winkler and Pasternak foundations have been studied by Paliwal et al. [2,3]. The results were a direct solution of the governing equations of motion and the distribution of the foundation was assumed to be uniform over the circumference. However, in practical applications, cylindrical shells are also embedded partially in an elastic foundation and the elastic foundation is not necessarily uniform in the longitudinal direction, resulting in a complicated problem. Free vibrations of a shell with a non-uniform elastic bed in the circumferential direction have been investigated by Amabili and Dalpiaz [4] based on the Rayleigh–Ritz method. Later on, complicating effects due to the contained inviscid fluid, elastic bed of partial axial and angular dimensions, intermediate constraint and added mass are considered by Amabili and Garziera [5]. In the investigation, the linear modes of simply supported shells vibrating in vacuo are used as admissible functions, and the solution is obtained with the artificial spring method. The admissible functions are the eigenfunctions of the closest, simple problem extracted from the one considered. So that, the problem extracted is "less constrained" than the original one [5]. Therefore, the formulation needs to be modified for problems "less constrained" than the simple supported shells.

This paper describes the free vibration characteristics of cylindrical shells considering a non-uniform distribution of elastic foundation in the circumferential and longitudinal directions. It is based on the finite element modeling of Timoshenko thin shell formulation [6], the shell is discretized into cylindrical finite elements to allow for the variation of foundation parameters in the longitudinal direction and the simple application of boundary conditions. Convergence issues of the present method were investigated. Variations in the natural frequency for various shell geometries and foundation parameters are presented. Finally, the paper presents applications of the method to analyze shells partially suspended by elastic foundations.

2. Formulation

The shell here is an isotropic thin elastic cylindrical shell with Young's modulus E, mass density ρ, Poisson's ratio υ, radius of the middle surface R, thickness h and length L. The foundation is represented by continuous elastic radial spring on a limited arc which corresponds to an angle $\varphi_1 + \varphi_2$. The geometry of the structure and generalized model with the reference directions are shown in Fig. 1, where K_w denotes the radial spring coefficient.

The displacements of a point on the middle surface in the axial, circumferential and radial directions are indicated by u, v and w. There are many available shape functions as listed in Cheung and Tham [7], the shape functions used here are the simplest and widely applied in many problems. The displacement functions can be expressed as

$$u(x,\theta) = \sum_{m=0}^{M} \{ U_m^S(x)\cos(m\theta) + U_m^U(x)\sin(m\theta) \},$$

$$v(x,\theta) = \sum_{m=0}^{M} \{ V_m^S(x)\sin(m\theta) + V_m^U(x)\cos(m\theta) \},$$

$$w(x,\theta) = \sum_{m=0}^{M} \{ W_m^S(x)\cos(m\theta) + W_m^U(x)\sin(m\theta) \}, \quad (1)$$

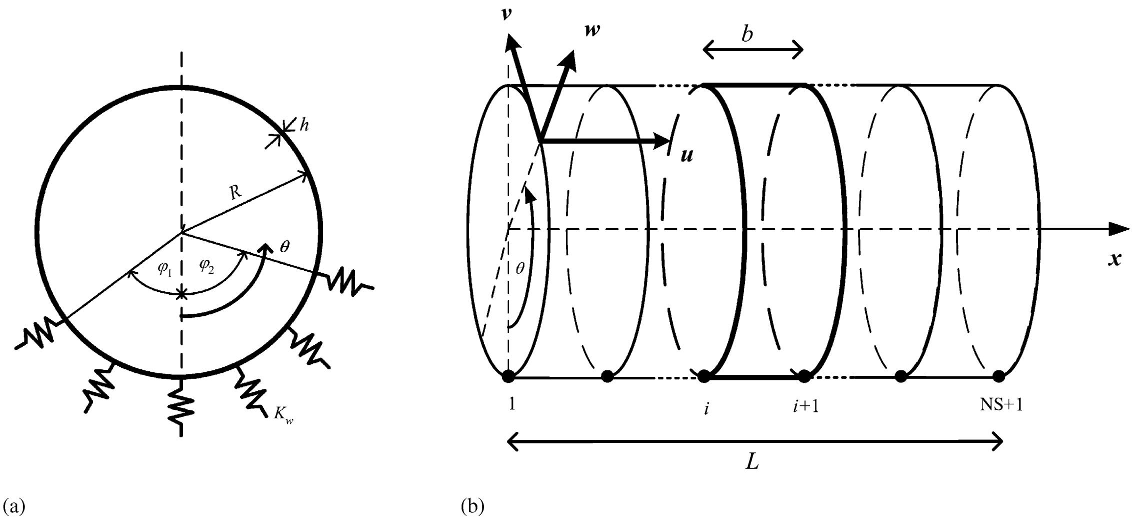

Fig. 1. Shell partially buried in an elastic foundation: (a) geometry and (b) generalized model.

in which linear polynomials in x are used for the axial and circumferential displacements, and a cubic polynomial is used for the radial displacement. The typical number of circumferential waves is denoted by m. Superscripts S and U refer to the symmetrical and asymmetrical deformations with respect to $\theta = 0$, respectively. By using the finite element method, stiffness ($\mathbf{K}_S$) and mass ($\mathbf{M}_S$) matrices of shells can be obtained as usual. The nodal parameters used here are the displacements and rotational angle β, which is defined as the first derivative of w with respect to x.

The distribution of radial spring along the circumferential direction may be expressed by the expansion of a Fourier series

$$\kappa_w(\theta) = \frac{K_w}{\pi}\left\{ a_0 + \sum_{l=1}^{\infty} [a_l \cos(l\theta) + b_l \sin(l\theta)] \right\}, \tag{2}$$

where a_l and b_l are the Fourier constants for a typical wavenumber l. The stiffness matrix of foundation for an element can be obtained by

$$\mathbf{K}_F = \int\int_A \mathbf{N}^{\mathrm{T}} \mathbf{\Psi} \mathbf{N}\, \mathrm{d}A, \tag{3}$$

where $\mathbf{N}$ is the shape function matrix and $\mathbf{\Psi} = \mathrm{diag}[0, 0, \kappa_w, 0]$.

In the calculation of Eq. (3), the constant term in Eq. (2) can be simply calculated under the trigonometric integrations; the cosine and sine terms in the same equation give non-zero values only when $m + n = l$ or $m - n = l$ ($m = 0, 1, 2, \ldots, M$ and $n = 0, 1, 2, \ldots, N = M$).

Based on the relations among m, n and l mentioned above and the necessity to define the foundation to be as smooth as possible, the total number of terms l should be taken as infinite. Therefore, the total number of circumferential waves M can be chosen independently as there is always a sufficient number of l to give values to the integrations developed by the matrix. The M value selected determines the total number of effective terms l in the truncated series.

Finally, the global equation of the problem is given by

$$\sum_{\mathrm{ns}=1}^{\mathrm{NS}}\sum_{n=0}^{N}\left[\begin{bmatrix}\mathbf{K}_{S_{mn}}^{SS} & 0\\ 0 & \mathbf{K}_{S_{mn}}^{UU}\end{bmatrix}+\begin{bmatrix}\mathbf{K}_{F_{mn}}^{SS} & \mathbf{K}_{F_{mn}}^{SU}\\ \mathbf{K}_{F_{mn}}^{US} & \mathbf{K}_{F_{mn}}^{UU}\end{bmatrix}\right]_{\mathrm{ns}}\begin{Bmatrix}\mathbf{de}_n^S\\ \mathbf{de}_n^U\end{Bmatrix}_{\mathrm{ns}}$$
$$=\omega^2\sum_{\mathrm{ns}=1}^{\mathrm{NS}}\sum_{n=0}^{N}\begin{bmatrix}\mathbf{M}_{S_{mn}}^{SS} & 0\\ 0 & \mathbf{M}_{S_{mn}}^{UU}\end{bmatrix}_{\mathrm{ns}}\begin{Bmatrix}\mathbf{de}_n^S\\ \mathbf{de}_n^U\end{Bmatrix}_{\mathrm{ns}}$$
$$\text{for } m=0,1,2,\ldots,N=M, \tag{4}$$

where ω is the natural frequency of the vibrating system, **de** is the nodal displacement parameter vector and NS is the total number of finite elements. The above equation is the standard generalized eigensystem and can be solved for the natural frequencies and modes. Note that, for the particular case where $\varphi_1=\varphi_2$, $\mathbf{K}_{F_{mn}}^{SU}$ and $\mathbf{K}_{F_{mn}}^{US}$ reduce to zero matrices and Eq. (4) splits into two separate problems (viz., symmetrical and asymmetrical vibrations).

3. Numerical results

This section investigates the convergence of the natural frequency as well as variations in the natural frequency and mode with different shell geometries and foundation parameters. A latter section also analyzes shells partially suspended by elastic foundations.

The paper considers two types of boundary conditions, simply supported (SS) and clamped at both ends (CC). For convenience, the non-dimensional frequencies in all the figures are expressed by $\Omega=\omega L\sqrt{\rho(1-\upsilon^2)/E}$. The enclosed angles are set to be equal ($\varphi_1=\varphi_2=\varphi$) for all numerical results presented here.

3.1. Convergence studies and validation

Numerical calculations were carried out to study the convergence of the natural frequency as a function of the total number of finite elements (NS) and the total number of circumferential waves (M). The following parameters were used in the calculation, $\upsilon=0.30$, $k_w(=K_wL/E)=0.001$ and $\varphi=\pi/3$. To illustrate the convergence behavior exhibited by different shell geometries, two types of shells are considered in this section. For convenience, these are referred to as follows: (i) Shell A, $R/L=0.5$, $R/h=200$ and (ii) Shell B, $R/L=0.05$, $R/h=20$.

Fig. 2 shows the convergence of Ω for the first symmetrical mode. As R/h increases, the M needed for convergence also increases. Increasing M to achieve the convergence of Shell A is shown as higher circumferential waves take part in the vibration.

Features related to the convergence studies can be summarized as follows:

(i) The results depend on the total number of finite elements (NS) and on the total number of circumferential waves (M).

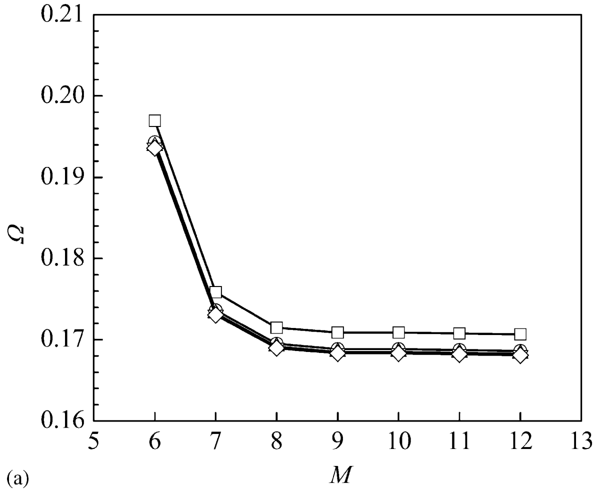

Fig. 2. Convergence of Ω for the first symmetrical mode (SS): (a) Shell A, $R/L = 0.5$, $R/h = 200$ and (b) Shell B, $R/L = 0.05$, $R/h = 20$. □, NS = 10; ○, NS = 20; △, NS = 30; ▽, NS = 40; ◇, NS = 50.

(ii) For Shell A, increase in M results in a significant contribution to the convergence of the results. For Shell B, NS has a strong influence on the results.
(iii) NS = 40 and $M = 20$ are necessary to assure the convergence of frequencies. Therefore, these parameters are used in the following analyses.

The next calculation compares the results with those of Amabili et al. [4]. The results are presented in Fig. 3 where the natural frequencies corresponding to the first four modes are plotted against the radial spring coefficient K_w. As can be seen from the comparisons, there is good agreement within the results. A maximum difference of about 1.5% is found for the result corresponding to the fourth mode and largest K_w.

3.2. Natural frequency of shells partially buried in elastic foundations

This section presents variations in the natural frequencies of shells with various geometries and foundation parameters.

Fig. 4 shows the variation of Ω with R/L for various values of R/h. The results for SS and CC are plotted in the figure. There is a significant increase in natural frequency for set values of R/L and R/h for CC due to the boundary effect. Generally, as R/L increases, the natural frequency decreases but in a fluctuating manner. These fluctuations are caused by changes in the sectional mode shape. The mode shapes of a shell with given parameters are plotted in Fig. 5 where the thickest line represents the enclosed arc. Corresponding frequency parameters are also given in the figure caption for SS, while the values in parenthesis correspond to frequency parameters for CC.

The effects of foundation parameters such as the radial spring stiffness and the enclosed angle play an important role in determining the characteristic of the vibrations of the whole system. For

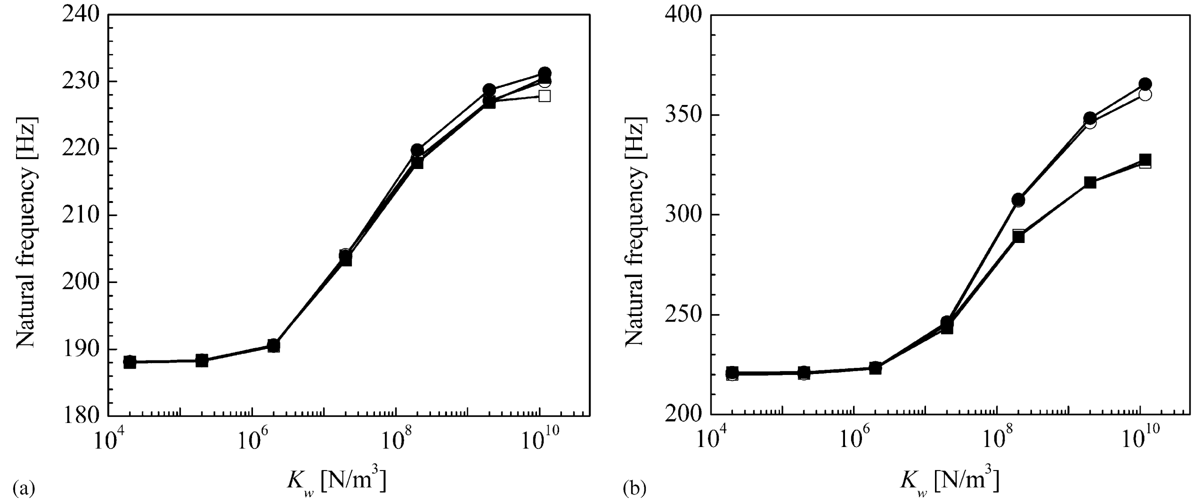

Fig. 3. Natural frequencies of the first four modes for shell with $v = 0.30$, $E = 206\,\text{GPa}$, $\rho = 7800\,\text{kg/m}^3$, $R = 300\,\text{mm}$, $L = 1000\,\text{mm}$, $h = 3\,\text{mm}$ and $\varphi = \pi/2$: (a) first two modes; (b) third and fourth modes. Solid symbols, present results; empty symbols, results obtained by Amabili and Dalpiaz [4].

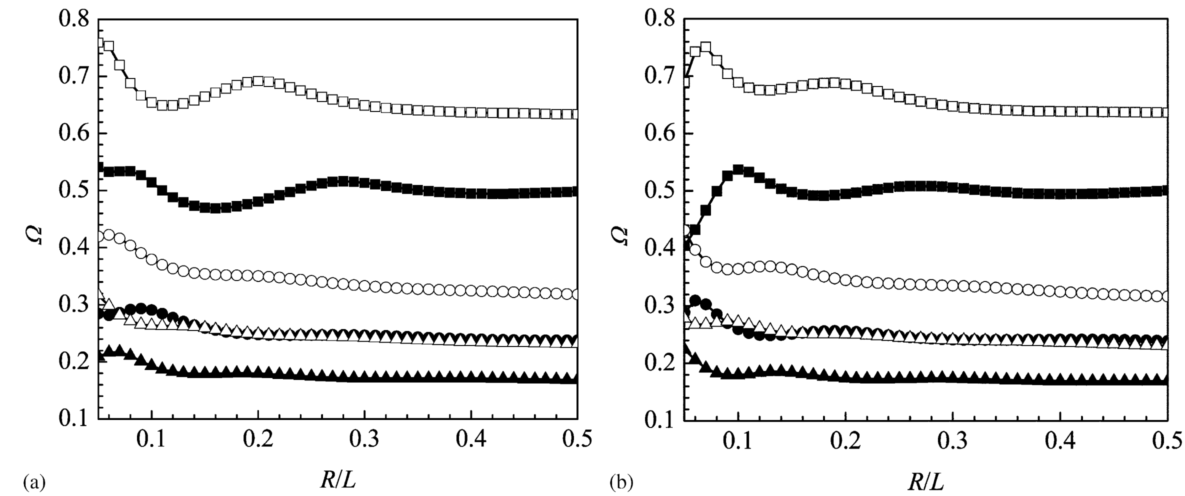

Fig. 4. Ω versus R/L for various values of R/h ($k_w = 0.003$ and $\varphi = \pi/3$): (a) first symmetrical mode; (b) first asymmetrical mode. ■, SS, $R/h = 20$; ●, SS, $R/h = 100$; ▲, SS, $R/h = 200$; □, CC, $R/h = 20$; ○, CC, $R/h = 100$; △, CC, $R/h = 200$.

the vibration system considered here, the relationship between the natural frequency and relative stiffness can be expressed as

$$\Omega^2 = (K_S^* + K_F^*)\big/M_S^* = \Omega_0^2(1 + \alpha). \tag{5}$$

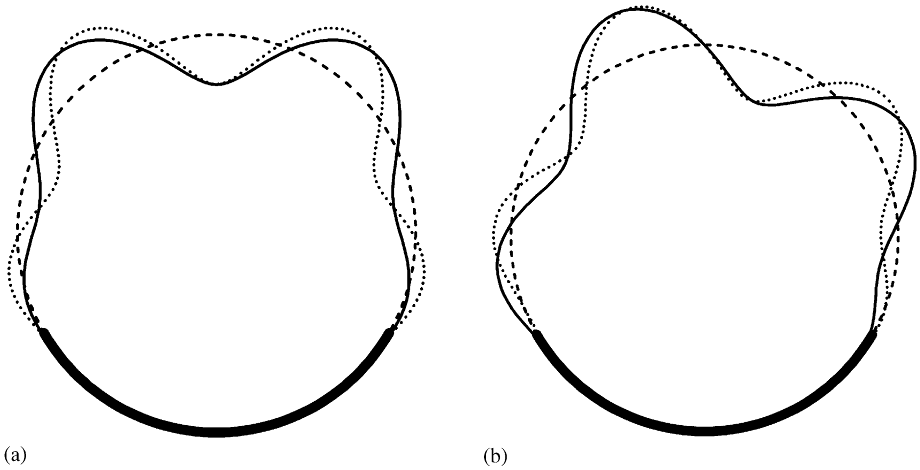

Fig. 5. Radial mode shapes of shell with $R/L = 0.20$, $R/h = 200$, $k_w = 0.003$, and $\varphi = \pi/3$: (a) first symmetrical mode $\Omega = 0.1798$ (0.2483) and (b) first asymmetrical mode $\Omega = 0.1741$ (0.2510). ———, SS; …………, CC.

The numerator in Eq. (5) includes the generalized stiffness of the shell, K_S^*, and the generalized stiffness of the foundation, K_F^*. The denominator is the generalized mass of the shell, M_S^*. The square of the frequency parameter for the shell in air, Ω_0^2, is equal to K_S^*/M_S^*; $\alpha = K_F^*/K_S^*$ is the relative stiffness ratio of the foundation and shell.

Fig. 6 shows α versus k_w for various values of φ. The α values of shells with small values of R/L and R/h are much larger than those of shells with large values of R/L and R/h and α becomes almost linear as φ increases. The relationship between α and k_w for shells with large values of R/L and R/h shows a plateau.

3.3. Shells partially suspended by elastic foundations

In some cases, the distribution of the foundation along the longitudinal direction may not be uniform, and in extreme cases, there is no foundation at some parts of shells. In this section, a shell simply supported at both ends and suspended partially across a gap between existing foundations is considered. The gap is located symmetrically about the midspan of the shell and the length of the gap is denoted by ηL, $\eta = 0$ and 1 correspond to the cases of the shell resting on an elastic foundation and the shell suspended in the air.

Fig. 7 shows the variations in Ω with η for various values of φ. The effect of the gap parameter η on the natural frequency is different for the cases shown in the figure. For shells with relatively small values of R/h, a larger η has a strong influence on the rate of decrement of Ω, especially at relatively small values of η.

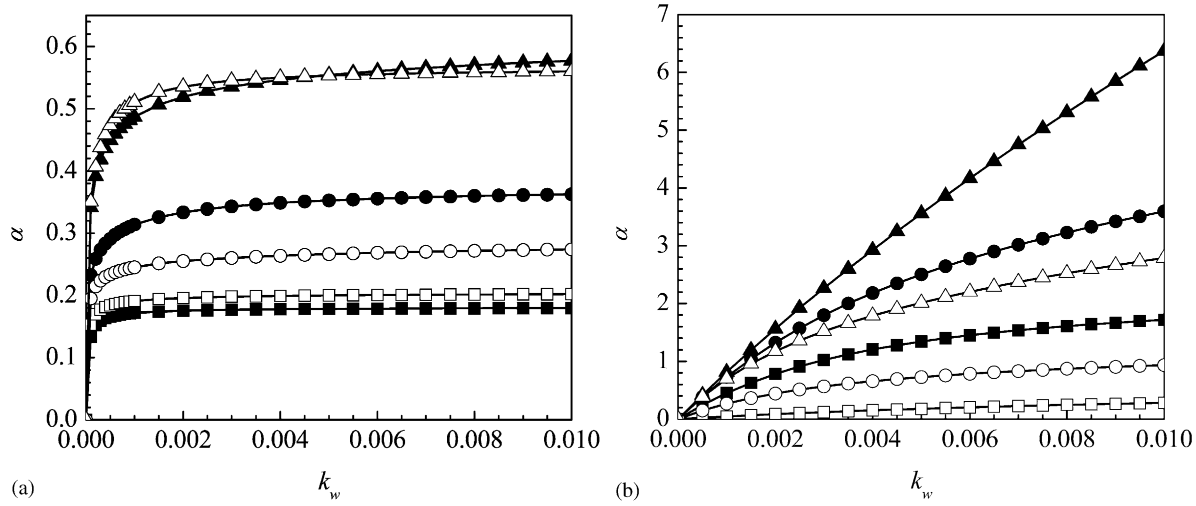

Fig. 6. α versus k_w for various values of φ (SS and first modes): (a) $R/L = 0.20$, $R/h = 200$, and $\Omega_0 = 0.1551$; (b) $R/L = 0.05$, $R/h = 20$, and $\Omega_0 = 0.3235$. ■, symmetrical, $\varphi = \pi/6$; ●, symmetrical, $\varphi = \pi/3$; ▲, symmetrical, $\varphi = \pi/2$; □, asymmetrical, $\varphi = \pi/6$; ○, asymmetrical, $\varphi = \pi/3$; △, asymmetrical, $\varphi = \pi/2$.

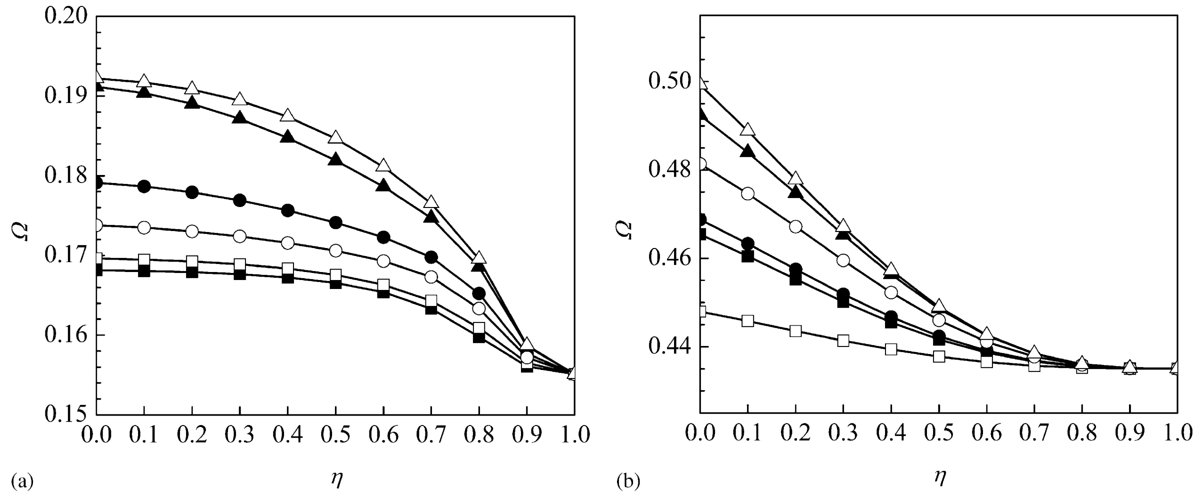

Fig. 7. Ω versus η for various values of φ (*SS*, first modes, $R/L = 0.20$ and $k_w = 0.002$): (a) $R/h = 200$; (b) $R/h = 20$. ■, symmetrical, $\varphi = \pi/6$; ●, symmetrical, $\varphi = \pi/3$; ▲, symmetrical, $\varphi = \pi/2$; □, asymmetrical, $\varphi = \pi/6$; ○, asymmetrical, $\varphi = \pi/3$; △, asymmetrical, $\varphi = \pi/2$.

Conclusions

Free vibration characteristics of cylindrical shells partially buried in elastic foundations are shown by means of the semi-analytical finite element method. The main conclusions of this study are summarized as follows:

(1) Non-uniformities of the foundation in the circumferential and longitudinal directions are simply taken into consideration in the analysis. Boundary conditions can be applied easily as in the finite element method.
(2) The fluctuation of Ω is observed as R/L increases. This is due to the gradual changes in the significant circumferential modes. However, the fluctuation diminishes as R/h increases.
(3) The relationship between the relative stiffness ratio α and k_w shows a plateau-like curve for shells with large values of R/L and R/h. For shells with small values of R/L and R/h, α correspond to symmetrical and asymmetrical modes are very much different.
(4) For shells with small values of R/L and R/h, both φ and k_w affect Ω significantly. However, for shells with large values of R/L and R/h on relatively stiff foundations, increasing k_w has no significant effect on Ω. In this case, φ is more pronounced.
(5) In the case of shells partially suspended by elastic foundations, a larger gap parameter η has a strong influence on the rate of decrement of Ω for shells with relatively small values of R/h, especially at relatively small values of η.

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